Endogenous fertility policy and unfunded pensions

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Abstract

We study the joint determination of fertility subsidies and social security taxes in an overlapping generations model where agents are heterogeneous in endowments. In equilibria where social security is valued, old and poor young agents form a coalition that sustains social security. When voting for fertility subsidies, the young take into account both the deadweight loss of such subsidies and the gains from a higher future tax base. They also take into account a third effect of increasing population growth: that of a decrease in future social security benefits as a consequence of a change in the identity of the future median voter.
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1 Introduction

In this paper we develop a political economy model of the joint evolution of unfunded social security (SS) systems and fertility policies. We use it to investigate the viability of unfunded social security and the role of fertility in securing it. We employ an overlapping-generations framework with endogenous fertility, in which agents vote on the size of fertility subsidies, on whether or not to continue with the SS system, and on the magnitude of pension taxes and payments. It is well known that a number of different policies - and in particular policies related to public education and immigration - can make SS viable in situations where otherwise it would be abandoned. However, our framework adds significantly to the literature by exploring the way in which current fertility subsidies can affect the identity of the future median voter, and thereby affect future pension/SS entitlements. This has not been investigated before. And yet as we discuss below there are clear cases in which actual fertility policies have been designed to alter the identity of future median voters.

While this strategic effect of fertility subsidies has received little attention in the context of the sustainability of SS, there is at least one example where such subsidies have been implemented to affect the identity of the future median voter. Winckler [2003] describes pro-natalist policies in Israel. From 1970 to 1996, family allowances were largely conditional on family members serving in the army, effectively discriminating against the Arab-Israeli population. Such policies were clearly motivated to maintain a 'demographic balance' between Jews and Arabs, as discussed in Portuguese [1998], Friedlander [1973], and expressed in the political arena most eloquently by David Ben Gurion (see Friedlander and Goldscheider [1979]).

In our model income heterogeneity serves two purposes. First, it allows us to investigate both within generation and between generation redistribution, which turns out to have important implications for our results. Second, it ensures that social security is valued even in the absence of dynamic efficiency, which we believe introduces a stronger motivation for the existence of the social security system. In this framework, fertility subsidies are valued only because they serve to obtain higher social security payments when old.

For our purposes, a number of recent contributions on the political economy of SS (see Galasso and Profeta [2002] for a survey) are especially relevant. First, Casamatta et al. [2000] and Tabellini [2000] study SS as a device that not only redistributes income from young to old, but also from wealthy to poor households, and therefore SS arrangements can be sustained as a political equilibrium without resorting to intergenerational (e.g. dynamic efficiency) considerations. Cooley and Soares [1999] and Galasso [1999] study SS as an institution with inherited rules that are costly to change, but their approaches are quite different from the one taken here. The paper that is closer to ours is Boldrin and Rustichini [2000]. In that model, agents are confronted with an existing promise of paying a SS tax to old agents, and may either abandon the SS system altogether, or pay the SS tax and vote for a level of SS benefits in the next period. The analysis in Boldrin and Rustichini [2000] gives an explicit dynamic dimension to the problem of sustaining a SS system, and provides a clear interpretation of it as one of unfunded SS liabilities, in line with most of the public policy debate.

A small number of studies examine the joint determination of SS along with another dimension of the welfare system, as we do. In particular, Conde-Ruiz and Galasso [2003] separate the redistributive aspect of SS into within and between cohort components, and consider the circumstances under which both aspects arise as an equilibrium. Boldrin and Montes [2002] show that unfunded SS creates incentives for the optimal provision of public education, a result generalized by Rangel [2003]. Finally, Kemnitz [2000] and Poutvaara [2003] study the joint determination of unfunded SS and education subsidies. We are aware of no paper that studies the joint determination of fertility policies and SS systems..

Our paper extends the literature on the political economy of SS systems to incorporate the endogenous determination of fertility policies. We find that strategic setting of fertility subsidies to limit the political influence of the newborn generation in the future decreases the levels of both fertility subsidies and SS taxes. We also obtain the result that the existence of fertility subsidies is a necessary condition for the existence of a unique, positive and globally stable steady state level of SS taxes.

This paper has four other sections. In the next section we present the model with the policy variables set exogenously. In section 3 we present the political economy environment, and define the equilibrium in which we are interested. In section 4 we derive the results with endogenous policy variables. Finally, section 5 summarizes our main conclusions.

2 The competitive equilibrium

We study an overlapping generations economy where households live for two periods. Households have preferences defined over their consumption when young c^y, the number of children to be born at the end of the first period n, and their consumption when old c^o. Labor supply is inelastic, with each household working one unit of time in period one and then retiring.

An endowment of α_i is received when young, and a storage technology allows saving at the rate $r = \frac{\beta}{1-\beta}$, where β is the time discount factor. Households are heterogeneous in their endowments (α), with the distribution of α over young households summarized by the uniform CDF $G(\alpha)$, with mean θ and support Θ:

$$G(\alpha_i) = \begin{cases} 0 & \text{if } \alpha_i < \underline{\alpha} \\ \frac{\alpha_i - \underline{\alpha}}{\overline{\alpha} - \underline{\alpha}} & \text{if } \alpha_i \in [\underline{\alpha}, \overline{\alpha}] \\ 1 & \text{if } \alpha_i > \overline{\alpha} \end{cases} \tag{1}$$

There are two policy instruments, SS taxes τ^{ss} and fertility subsidies τ^f. The SS system is unfunded, or pay-as-you-go, so young households pay a proportional tax on their endowment, which finances the unique level of SS benefits received by old households at every period. Fertility subsidies (taxes) are designed to reduce (increase) the cost of having children bn, where b is the cost per child, and are financed by a lump sum tax (rebate) T. Note that n is normalized so that the unit of measurement of the population is the young (two person) household, so $n = 1$ implies 2 children per couple. Household i solves the problem

$$\begin{aligned} \max_{\{c^y, c^o, n\}} \quad & c_t^y + \beta c_{t+1}^o \quad + \gamma \ln n_t \\ s.t. \quad & (1 - \tau_t^{ss})\alpha_i = s_t + c_t^y + b(1 - \tau_t^f)n_t + T_t \\ & c_{t+1}^o = s_t(1 + r) + ss_{t+1}. \end{aligned} \tag{2}$$

Here, s_t is savings, and ss_{t+1} represents SS benefits at time $t + 1$. The use of quasilinear preferences, together with a lower bound on individual endowments α_i will ensure that the choice of n is independent of wealth, which simplifies aggregation greatly.

In this arrangement, all households choose the same number of children n_t, and the consumption path c^y/c^o for any household is indeterminate. Fertility and total consumption follow [1]

$$n_t = \frac{\gamma}{b(1-\tau_t^f)}. \quad (3)$$

$$c_t^y + \beta c_{t+1}^o = (1-\tau_t^{ss})\alpha_i + ss_{t+1}/(1+r) - T_t - \gamma. \quad (4)$$

The government raises taxes and provides subsidies and SS benefits under a restrictive rule of budget balance: SS benefits are financed with SS contributions, and fertility subsidies are financed through a lump sum tax paid only by the young. These conditions are formalized below.

$$N_{t+1}\tau_{t+1}^{ss}\theta = N_t ss_{t+1} \quad (5)$$

$$b n_t \tau_t^f = T_t \quad (6)$$

As ss_{t+1} is the same for every old household, but SS taxes are paid according to earnings, the SS system is redistributive. Because endowments are exogenous on the other hand, SS is not distortionary. Fertility subsidies on the contrary are distortionary, but do not imply a redistribution of income. The redistributive aspect of SS and the distortionary aspect of fertility subsidies can therefore be isolated in this setting. The model is closed by specifying the law of motion for population. The working population (N_t) evolves according to

$$N_{t+1} = n_t N_t. \quad (7)$$

We are now ready to define an equilibrium.

Competitive equilibrium: A competitive equilibrium is a collection of optimal policies $\{c_{i,t}^y, c_{i,t}^o, n_t\}$, which, together with a sequence of tax and benefit levels $\{\tau_t^f, \tau_{t+1}^{ss}, T_t, ss_t\}_{t=0}^{\infty}$, and a distribution of types over households $G(\alpha_i)$, satisfy:

1. Given taxes/benefits, the optimal policies $\{c_{i,t}^y, c_{i,t+1}^o, n_{i,t}\}$ solve the household problem.

2. The government budget is balanced according to (5) and (6).

In the next section, we present the political mechanism used to determine the levels of tax rates at every period.

[1] We restrict the parameters to ensure an interior solution where consumption is strictly positive. The condition on the parameters is that disposable income is larger than γ:$\gamma < ss_{t+1}/(1+r) + (1-\tau_t^{ss})\underline{\alpha} - T_t$

3 Voting over taxes

We now extend the framework to allow for the determination of whether the SS system is abandoned or continued and, if continued, the levels of fertility subsidies and SS taxes. In our majority voting model, such decisions are set to maximize the utility of the median-voter household. This arrangement can be thought of as a game where successive cohorts of players play the game twice, once when young and again when old.

Players: Since players are indexed by their date of birth t and endowment α_i, we can represent the set of households at date j as

$$P(j) = \{(i,t) \in \mathbb{R}^2 : i \in \Theta, t \in (j, j-1)\}$$

To avoid confusion between the date the game is played and the date the players are born, we will also use $\{y, o\}$ to denote young (born in the current date), and old (born in the previous date) players.

Actions and Timing of decisions: At each period t, young households are confronted with a SS tax τ_t^{ss}, promised to the current old at $t-1$ and earmarked to finance a unique level of retirement benefits. Young and old households then vote over three policy dimensions.

First, households vote on λ_t, expressing their preference for honoring the promise τ_t^{ss} or defaulting. If a majority of voters chooses to default, young households lose their right to obtain SS payments next period, so a trigger strategy sustains the equilibrium. Alternatively voters may choose to continue SS. Households then proceed to vote for their preferred levels of future SS tax τ_{t+1}^{ss} and current level of fertility subsidy τ_t^f.

Because they die at the end of the period, old households are indifferent as to the levels of SS taxes or population levels in the future. We assume that, when policy choices do not affect voters payoffs, they abstain from voting. Abstention then needs to be included as a possibility in the set of actions a_t.

$$a_t = \{\lambda_t, \tau_{t+1}^{ss}, \tau_t^f\}$$

where $\lambda_t \in \{1, 0\}$ is a vote for keeping (1) or rejecting (0) SS, and we use λ_t^m to denote the median voter; $\tau_{t+1}^{ss} \in [0,1] \cup \{abst\}$ and $\tau_t^f \in (-\infty, 1) \cup \{abst\}$. The timing of these decisions is illustrated in figure 1.

We follow the literature in assuming sincere voting: Households vote for their preferred choices even if this will not change the equilibrium outcomes.

Payoffs: To obtain a derived preference ordering over tax rates for young households, we solve for the value function in problem (2). For young household i this function is

$$V_i^y(\tau_t^f, \tau_{t+1}^{ss}, \tau_t^{ss}) = (1-\tau_t^{ss})\alpha_i + \frac{\theta\beta\gamma\tau_{t+1}^{ss}}{(1-\tau_t^f)b} - \frac{\gamma}{1-\tau_t^f} + \gamma \ln \frac{\gamma}{(1-\tau_t^f)b}. \quad (8)$$

Equation (8) shows that young household i's utility is increasing in the endowment α_i, aggregate level of endowments θ, and next period SS tax rate τ_{t+1}^{ss}, and decreasing in the current period's SS tax τ_t^{ss}.

Young households first cast a vote on rejecting versus keeping the SS system. If a majority of voters (young and old) choose to abandon the SS system ($\lambda^m = 0$), the payoff for all young households is $V_i^y(0,0,0)$[2]. If the majority choose to keep SS, young households pay τ_t^{ss} of their endowment, vote on $\{\tau_t^f, \tau_{t+1}^{ss}\}$, and proceed to vote on λ_{t+1} in the next period. If $\lambda_{t+1}^m = 1$, so that SS is continued at $t+1$, the payoff is $V_i(\tau_t^f, \tau_{t+1}^{ss}, \tau_t^{ss})$, but if SS is abandoned at $t+1$, the payoff for the young at t is $V_i(\tau_t^f, 0, \tau_t^{ss})$, as the young households pay contributions at t but do not receive benefits at $t+1$. Summarizing, the payoff function for the young at time t is

$$\pi_{y,t} = \begin{cases} V_i(\tau_t^f, \tau_{t+1}^{ss}, \tau_t^{ss}) & \text{if } \lambda_t^m = 1 \text{ and } \lambda_{t+1}^m = 1 \\ V_i(\tau_t^f, 0, \tau_t^{ss}) & \text{if } \lambda_t^m = 1 \text{ and } \lambda_{t+1}^m = 0 \\ V_i^y(0,0,0) & \text{if } \lambda_t^m = 0 \end{cases} \quad (9)$$

For old households note that, since the consumption path c^y/c^o is indeterminate, their value function is also indeterminate. For our purposes, it will be useful to take the level of individual saving as given (and equal to zero without loss of generality), and consider the function

$$V^o(\tau_t^{ss}) = \frac{\theta w \tau_t^{ss}\gamma}{(1-\tau_{t-1}^f)b}. \quad (10)$$

At time t, old households prefer higher levels of τ_t^{ss}, but are indifferent among different levels of tax rates $\{\tau_{t+1}^{ss}, \tau_t^f\}$. The payoff function for old households maps the choice of λ by the median voter, to the value function $V^o(\tau_t^{ss})$ in (10):

$$\pi_{o,t} = \begin{cases} V^o(\tau_t^{ss}) & \text{if } \lambda_t^m = 1 \\ 0 & \text{if } \lambda_t^m = 0 \end{cases} \quad (11)$$

[2]Here we impose the equilibrium result that if SS is abandoned, households have no incentives to choose levels of fertility subsidies different from zero

Initial conditions: Initial conditions in this model are an initial level of SS tax τ_0^{ss}. This completes the description of the game, now we just need an equilibrium concept.

Games of the type studied here are usually subject to indeterminacy of the equilibrium. We solve this problem by focusing on Markov Perfect Equilibria: We look for policy functions for the voting choices that are functions of the current state of the economy, summarized by the current SS tax. In addition, we impose the condition that the allocation is a competitive equilibrium given tax rates.

Voting Equilibrium: A voting Equilibrium is a collection of policy functions $\{\tau^{ss}(\tau_t^{ss};\alpha_i,y),\tau^{ss}(\tau_t^{ss};\alpha_i,o),\tau^f(\tau_t^{ss};\alpha_i,y),\tau^f(\tau_t^{ss};\alpha_i,o),\lambda(\tau_t^{ss};\alpha_i,y),\lambda(\tau_t^{ss};\alpha_i,o)\}$, for young and old voters, optimal policies for consumption and fertility $\{n_t,c_t^y,c_t^o\}$ and the n-tuple $\{r,w,N_t,G(\alpha)\}$, that satisfy:

1. The choice of $\{\lambda_t,\tau_{t+1}^{ss},\tau_t^f\}=\{\lambda(\tau_t^{ss};\alpha_i,.),\tau^{ss}(\tau_t^{ss};\alpha_i,.),\tau^f(\tau_t^{ss};\alpha_i,.)\}$ by household i at time t maximizes his payoff, given that all other households play the equilibrium policies at time periods $0,1,...,t-1,t+1,...$

2. Given $\{\tau_t^{ss},\tau_t^f,\lambda_t^m\}_{t=0}^{\infty}$, consumption and fertility decisions and the n-tuple $\{r,w,N_t,G(\alpha)\}$ form a competitive equilibrium.

4 Properties of the equilibrium

In this section we examine the properties of the voting equilibrium. In particular, we illustrate the two main tradeoffs present in the voting model. The first tradeoff involves the gains from increased fertility in the form of a larger tax base in the future, weighted against the deadweight loss from subsidizing fertility. The second tradeoff involves weighting the net gains from increased fertility via a larger tax base, as discussed above, against a lower level of SS taxes that tomorrow's median voter will be willing to pay.

We begin by characterizing voting by young and old households [3].

[3]One possible equilibrium of this game has the young at t voting for fertility subsidies/taxes that imply $n_t=1$, ensuring that they will constitute a majority when old in period $t+1$, and at the same time setting $\tau_{t+1}^{ss}=1$, so that all the income is extracted from the young at $t+1$. At $t+1$ the economy ends. We do not think that this is an interesting equilibrium, and in this section we will restrict the parameters of the model to eliminate it from the analysis. The propositions that follow refer then to an equilibrium where $n_t>1$ for all t.

Proposition 1 *Voting behavior with SS.*

1. *The median voter ($\alpha^i = \alpha^m$) is indifferent between the allocation with and without SS: Half of the voters choose $\lambda = 1$ and the other half $\lambda = 0$.*

2. *The old vote for continuation of the SS system ($\lambda = 1$) but are indifferent as to the levels of $\{\tau^{ss}_{t+1}, \tau^f_t\}$, so they abstain from voting over taxes.*

3. *There exists an endowment level α^m such that the young vote $\lambda^i_t = 1$ if $\alpha^i \leq \alpha^m$ and $\lambda^i_t = 0$ otherwise.*

4. *All (young) voters choose the same tax rates.*

Proof: For point 1, note that if the median voter at $t+1$ receives a net surplus from continuing SS, the median voter at t could have increased τ^{ss}_{t+1} by a small amount, and SS would still be continued at $t+1$. On the other hand, if the median voter at $t+1$ receives a net loss from continuing SS, he will vote $\lambda^m = 0$, so the tax rates chosen by the median voter at t could not have been derived from equilibrium policies.

For point 2, note that the payoff of the old (expression 10) is higher if $\lambda^m = 1$. Because of sincere voting, old households will choose $\lambda = 1$. On the other hand, tax rates are not arguments of this payoff function, so the old abstain from voting over taxes.

To prove the third point, note that the value of keeping the SS promise for young household i is

$$V_i(.,.,.) - V_i(0,0,0) = -\tau^{ss}_t \alpha_i + \frac{\theta\beta\gamma\tau^{ss}_{t+1}}{(1-\tau^f_t)b} - \frac{\gamma\tau^f_t}{1-\tau^f_t} - \gamma\ln(1-\tau^f_t) \quad (12)$$

Which is decreasing in the endowment α_i and is negative for all endowments larger than some critical value α^m. This implies that the SS system is always sustained by a coalition of the old (who are always beneficiaries) and the poor young.

For point 4, note that because V^y_i is separable in α_i, the problem of choosing tax rates once SS is continued can be stated without α_i being an argument of the objective function or the constraints (see appendix A.1).

In the current framework, the only motivation to vote for fertility subsidies comes from the existence of unfunded SS. At the same time, it is the heterogeneity in endowments that makes social security valued in equilibria that are dynamically efficient. The following lemma formalizes these results.

Lemma 1 *Interactions between SS and fertility subsidies.*

1. *Social security is necessary for fertility subsidies to be valued*

2. *If the economy is dynamically efficient, heterogeneity in endowments is necessary for social security to be valued.*

Proof: For point 1, the net value of voting for fertility subsidies in the absence of social security (from expression 12) is

$$-\frac{\gamma \tau_t^f}{1-\tau_t^f} - \gamma \ln(1-\tau_t^f) \tag{13}$$

with a maximum at $\tau_t^f = 0$.

For point 2, note that if the economy is dynamically efficient the equilibrium without taxes is Pareto optimal, so any gains from SS to the median voter are obtained through redistribution.

In solving for the Markov Perfect Equilibrium we use the fact that the median voter is indifferent between continuing or abandoning SS (point 1 of proposition 1). Households then choose tax rates $\{\tau_{t+1}^{ss}, \tau_t^f\}$ by maximizing their indirect utility subject to the constraint that the median voter next period will be indifferent in his choice over λ. This effectively implies that the voters at t must know the realization of τ_{t+2}^{ss} when choosing τ_{t+1}^{ss}. In this paper, we will use the following interpretation of this paradox: The institution of SS makes a promise of tax rates into the future, and voters take this promise as their expected value of future SS tax levels. This interpretation will allow for a discussion of the role played by expected future benefits in the sustainability of SS. In equilibrium such expected values are self fulfilling. An expected SS tax rate will be denoted with a hat (e.g. $\widehat{\tau}_{t+2j}^{ss}$).

We present the Markov Perfect Equilibrium in three steps. First, we derive the evolution of SS in an economy without fertility subsidies. Then, we introduce fertility subsidies but examine the outcomes when agents do not internalize the effects of their choices on the identity of the future median voter. Finally, we study the model with fully rational agents.

4.1 No fertility subsidies

It is instructive to consider initially the model with no recourse to fertility subsidies. We proceed backwards by first deriving the tax rates chosen if SS is not abandoned, and then considering the choice of keeping the SS system. Young households choose $\{\tau_{t+1}^{ss}\}$ by solving a problem that takes the form

$$\max_{\tau_{t+1}^{ss}} \quad (1-\tau_t^{ss})\alpha_i + \tfrac{\theta\beta\gamma\tau_{t+1}^{ss}}{b} - \gamma \quad +\gamma \ln \frac{\gamma}{b} \tag{14}$$

$$s.t. \quad -\tau_{t+1}^{ss}\alpha_m + \tfrac{\theta\beta\gamma\widehat{\tau}_{t+2}^{ss}}{b} \geq 0 \quad \text{if } \gamma/b > 1 \tag{15}$$

$$\tau_{t+1}^{ss} \leq 1 \quad \text{otherwise.} \tag{16}$$

With $\alpha_m = \underline{\alpha} + (\overline{\alpha} - \underline{\alpha})\frac{\gamma/b-1}{2\gamma/b}$. Constraint (16) represents the fact that, if $n_t = \gamma/b \leq 1$, the young can set $\tau_{t+1}^{ss} = 1$, and the economy ends. In what follows we disregard this possibility and consider only interior solutions, where constraint (15) applies. The optimal choice for the SS tax rate is

$$\tau_{t+1}^{ss} = \frac{\theta}{\alpha_m}\frac{\gamma}{b}\beta\widehat{\tau}_{t+2}^{ss} \tag{17}$$

We can decompose the coefficient multiplying $\widehat{\tau}_{t+2}^{ss}$ into three parts: $\frac{\theta}{\alpha_m}$, $\frac{\gamma}{b}$, and β. Note that the ratio of average income to the income of the richest household voting for SS, denoted by $\theta/\alpha_m > 1$, is a measure of how redistributive the SS system is. Note also that the ratio γ/b is both the fertility rate and the dependency ratio in the next period. The intuition behind (17) is as follows: The larger θ/α_m is, the more do low endowment young households at $t+1$ have to gain from SS for any given tax rate $\widehat{\tau}_{t+2}^{ss}$. Consequently, they in turn can be taxed more and still want to preserve SS. A similar argument holds for the fertility rate, as a higher rate implies larger SS benefits for any given tax rate, since there are more young households to be taxed. Finally, since costs τ_{t+1}^{ss} are paid one period before SS benefits for the median voter at $t+1$, these benefits need to be discounted by β.

The policy function for SS taxes can be obtained by inverting (17), using the fact that, in equilibrium, $\{\widehat{\tau}_{t+2}^{ss}\}_{t=0}^{\infty} = \{\tau_{t+2}^{ss}\}_{t=0}^{\infty}$.

$$\tau_{t+1}^{ss} = \tau_t^{ss}\frac{\alpha_m b}{\theta\beta\gamma} \tag{18}$$

Which defines the MPE for this model. Since the law of motion for SS taxes is linear, in general an interior steady state will not exist. This is formalized in the next lemma

Lemma 2 *In an economy without fertility subsidies, a stable interior steady state for social security taxes exists only in the case* $\frac{\alpha_m b}{\theta\beta\gamma} = 1$.

Proof: Without fertility taxes, the dynamics of the system depend on $x \equiv \frac{\alpha_m b}{\theta\beta\gamma}$. *If* $x > 1$, *no positive levels of SS benefits can be sustained as part of an equilibrium, as the economy reaches a point where tax rates higher than one are necessary to sustain the system. If* $x = 1$, *the economy is stable at any positive level of* τ^{ss} , *and if* $x < 1$, *SS tax rates converge monotonically to zero, the only steady state level of SS taxes.*

This result is similar to those in Boldrin and Rustichini [2000], who find that zero is the only stable steady state, and that therefore any SS system will gradually shrink as time progresses.

In this version of our model, a high fertility rate, high inequality, and high degree of patience (β close to one), all contribute to making SS implementable ($x < 1$) since they imply that any level of future SS taxes is associated with higher levels of benefits. Median voters may therefore prefer to continue the SS system even if the sequence of SS tax rates is decreasing.

4.2 Fertility subsidies

We now allow for voting over fertility subsidies. We begin by considering the choice of $\{\tau_t^f, \tau_{t+1}^{ss}\}$, conditional on the SS system being continued. Using the expression for α_m in (22), and with n_t given by (3), the problem of choosing $\{\tau_t^f, \tau_{t+1}^{ss}\}$ for young households becomes:

$$\max_{\{\tau_t^f, \tau_{t+1}^{ss}\}} \quad (1-\tau_t^{ss})\alpha_i + \tfrac{\theta\beta\gamma\tau_{t+1}^{ss}}{(1-\tau_t^f)b} - \tfrac{\gamma}{1-\tau_t^f} \quad + \gamma \ln \frac{\gamma}{(1-\tau_t^f)b} \tag{19}$$

$$s.t. \quad -\tau_{t+1}^{ss}\{\tfrac{(1-(1-\tau_t^f)b/\gamma)(\overline{\alpha}-\underline{\alpha})}{2} + \underline{\alpha}\} + \tfrac{\theta\beta\gamma\widehat{\tau}_{t+2}^{ss}}{(1-\tau_{t+1}^f)b} - \tfrac{\gamma}{1-\tau_{t+1}^f} + \gamma + \gamma \ln \tfrac{1}{1-\tau_{t+1}^f} = 0 \quad \text{if } \tau_t^f > 1-\gamma/b \tag{20}$$

$$\tau_{t+1}^{ss} \leq 1 \quad \text{otherwise} \tag{21}$$

Figure 2 illustrates the problem of choosing $\{\tau_{t+1}^{ss}, \tau_t^f\}$, where the first part of the constraint (equation (20)) has been rearranged to highlight that τ_{t+1}^{ss} is a function of $\widehat{\tau}_{t+2}^{ss}$. The problem has an interior solution only for $\widehat{\tau}_{t+2}^{ss}$ above a critical value. In the figure, $\widehat{\tau'}_{t+2}^{ss}$ is above this critical value, but $\widehat{\tau}_{t+2}^{ss}$ is not.

This problem is equivalent to (14)-(16) except that the identity of the median voter is now a function of fertility choices in the previous period, as shown in the following lemma.

Lemma 3 *Higher (lower) fertility subsidies at t imply a higher (lower) endowment median voter at $t+1$.*

Proof: Note that, by point 2 of proposition 1, old voters will unanimously choose to honor the SS promise ($\lambda_{t+1} = 1$), since they are the beneficiaries. If we normalize the number of old households to 1, then the mass of voting households is $1 + n_t$. With $n_t > 1$, the median-voter household is such that a proportion $\frac{n_t - 1}{2n_t}$ of young households will vote $\lambda_{t+1} = 1$ [4]. Together with the fact that the poorest young households are the ones to vote for continuing SS, which follows from point 3 of proposition 1, this implies that the median voter household at time $t+1$ will have an endowment level

$$\alpha_{m,t+1} = \underline{\alpha} + (\overline{\alpha} - \underline{\alpha})\frac{n_t - 1}{2n_t}. \tag{22}$$

By (3), we have $\frac{\partial n}{\partial \tau^f} > 0$, so $\frac{\alpha_{m,t+1}}{\tau_t^f} > 0$

This result is illustrated in figure 3, where the endowment level is measured on the vertical axis. The horizontal axis represents the number of voters, where the number of old households is normalized to one and young households are ranked from poorest to wealthiest. As fertility increases from n to n', the endowment level of the median voter household increases from α_m to α'_m.

We discuss the MPE of this model in two steps, always assuming an interior solution. First, we derive the equilibrium in a model where agents do not anticipate the effects of their actions on the identity of the future median voter. This myopic version of the equilibrium will allow for a discussion of the first tradeoff in the choice of fertility subsidies: that of a higher tax base and therefore larger future SS benefits, against the deadweight loss of the subsidy. We then discuss the equilibrium in the model with fully rational voters, which will allow for a discussion of the second effect: that of lower future SS benefits due to a change in the identity of the median voter.

[4] If $n_t > 1$, the median voter household for the two generations is such that $(1 + n_t)/2$ vote for the same choice. Thus the proportion of the young voting the same way as the old is given by $[\frac{1+n_t}{2} - 1]/n_t = \frac{n_t - 1}{2n_t}$.

4.2.1 Myopic voters

In this subsection we study a MPE with myopic voters. In an interior solution, with μ denoting the multiplier assigned to constraint (20), tax rates are obtained from the following first order conditions:

$$(\tau_{t+1}^{ss}) \quad \tfrac{\theta\beta\gamma}{(1-\tau_t^f)b} - \mu\{\underline{\alpha} + \tfrac{(\overline{\alpha}-\underline{\alpha})}{2}(1-(1-\tau_t^f)b/\gamma)\} = 0 \tag{23}$$

$$(\tau_t^f) \qquad -\tfrac{\gamma}{(1-\tau_t^f)^2} + \tfrac{\theta\beta\gamma\tau_{t+1}^{ss}}{b(1-\tau_t^f)^2} + \tfrac{\gamma}{(1-\tau_t^f)} = 0 \tag{24}$$

$$\begin{aligned}(\mu) \qquad & -\tau_{t+1}^{ss}\{\underline{\alpha} + \tfrac{(\overline{\alpha}-\underline{\alpha})}{2}(1-(1-\tau_t^f)\tfrac{b}{\gamma})\} \\ & + \tfrac{\theta\beta\gamma\widehat{\tau}_{t+2}^{ss}}{(1-\tau_{t+1}^f)b} - \tfrac{\gamma}{1-\tau_{t+1}^f} + \gamma + \gamma\ln\tfrac{1}{1-\tau_{t+1}^f} = 0\end{aligned} \tag{25}$$

The first condition is actually redundant in determining the equilibrium, which can be obtained from equations (24) and (25).

Equation (24) governs the choice of τ_t^f: the first term represents the cost of paying for higher subsidy levels for each birth, and the third represents the utility gains from having more children. Together these terms are the (net) deadweight loss from subsidizing fertility, and the myopic voter weights this loss against the utility gain from higher future SS benefit levels, represented by the second term.

The last condition, equation (25), is the incentive compatibility condition, and says that tomorrow's voter must be at least indifferent between keeping and abandoning SS, given the chosen tax rates.

The MPE for this model is given by

$$\tau_{t+1}^{ss} = \frac{b}{\theta\beta}\{1-\exp\{-\frac{\tau_t^{ss}\widehat{\alpha_{m,t}}}{\gamma}\}\} \tag{26}$$

$$\tau_t^f = 1-\exp\{-\frac{\tau_t^{ss}\widehat{\alpha_{m,t}}}{\gamma}\} \tag{27}$$

Where $\widehat{\alpha_{m,t}}$ is an equilibrium object that maps the current SS tax to the current median voter:

$$\widehat{\alpha_{m,t}} \equiv \underline{\alpha} + \frac{(\overline{\alpha}-\underline{\alpha})}{2}(1-(1-\frac{\theta\beta\tau_t^{SS}}{b})b/\gamma) \tag{28}$$

Equation (26) describes the equilibrium sequence of SS taxes (see appendix A.2 for the derivation). Because $\frac{b}{\theta\beta}$ may be larger than one, the sequence in (26) may reach tax rates higher than one in finite time, in which case no equilibrium exists.

4.2.2 Rational voters

We now consider the model with voters who anticipate the effect of their choice of τ^f on the identity of the future median voter. The equilibrium is the solution to the same first order conditions for τ^{ss}_{t+1} and μ in the previous version (equations (23) and (25)), plus the following first order condition that governs the choice of τ^f_t:

$$-\frac{\gamma}{(1-\tau^f_t)^2}+\frac{\theta\beta\gamma\tau^{ss}_{t+1}}{b(1-\tau^f_t)^2}+\frac{\gamma}{(1-\tau^f_t)}-\mu\tau^{ss}_{t+1}\frac{b}{\gamma}\frac{(\overline{\alpha}-\underline{\alpha})}{2}=0 \qquad (29)$$

Note that the first three terms are the same as in the previous example (equation (24)). The fourth term adds a further important effect from increasing τ^f_t. As fertility in one period increases, the endowment level of tomorrow's median voter household will also increase. Because the young now form a larger constituency, a larger proportion of them is needed to form a majority pro-SS together with the old. By Lemma 3, the median voter will now be a household with a higher endowment level than before. Because SS is redistributive, a higher endowment median-voter household obtains lower net gains from participating in SS, so she will be indifferent between maintaining or abandoning the SS system at a *lower* SS tax τ^{ss}_{t+1} for each promised tax rate $\widehat{\tau}^{ss}_{t+2}$.

Starting from a common initial condition, accounting for this effect implies that both equilibrium fertility subsidies and SS taxes are lower in the economy with rational voters than in the economy with myopic voters. For fertility subsidies, the rational voter anticipates the negative effect on welfare of higher population growth, so the result is intuitive. For social security taxes, we can provide the following intuition: by construction, a median voter who is rational obtains higher utility benefits from choosing fertility, so she will need a lower future SS tax in order to become indifferent between keeping or abandoning the SS system.

Because a closed form solution for the MPE cannot be obtained in the case of rational voters, we compare the equilibria for the three models using numerical simulations. We choose two sets of plausible parameter values to illustrate what we believe are the interesting equilibria. [5] The laws of motion

[5] We choose the following parameter values for Example 1: $\beta = .15$, which implies an annual interest rate of 6.5% for 30 years. $b = .12$ and $\gamma = .56$, which implies a dependency ratio $\frac{\gamma}{b}$ of 4.7 in the absence of fertility subsidies, and $\underline{\alpha} = .4$, $\overline{\alpha} = 2.6$ for the distribution of endowments. Example 2 increases γ to .8

of SS taxes and fertility subsidies are plotted in figure 4. Figures 4A and B represent an equilibrium where SS is not viable without fertility subsidies. Figure 4A shows the law of motion for SS taxes against a forty-five degree line. The horizontal axis represents SS taxes at time t, and the vertical axis represents SS taxes at $t + 1$. The crossed line shows the law of motion for the model without fertility subsidies. For these parameters, SS taxes reach values higher than one in finite time, so no SS system can be implemented. The dotted and continuous lines show the laws of motion for the models with myopic and rational voters respectively. Note that in both cases the SS tax converges to an interior steady state.

Figure 4B shows the dynamics of fertility subsidies for the economy with myopic voters (dotted line) and rational voters (full line) against a forty-five degree line. The horizontal axis again represents SS taxes at time t, the state variable, and the vertical axis represents the rate of fertility subsidies. Note that the strategic effect on the identity of tomorrow's median voter implies that chosen fertility subsidies are lower for rational voters, as expected.

Figures 4C and D show the same dynamics for an example where SS is valued in the absence of fertility subsidies. In this case, the only steady state for all three economies is zero, and both SS and fertility subsidy systems shrink with time. [6]

5 Conclusion

In this paper, we developed a political economy model of social security and fertility subsidies, where young generations confront promises made previously by older generations and in turn promise themselves future levels of SS benefits. A trigger strategy, with the threat of abandoning SS, sustains the voting equilibria where SS is valued.

We highlighted that, when choosing to subsidize fertility, young generations not only increase the tax base in the future but at the same time also limit their own political influence. Strategic setting of subsidies to fertility to change the identity of the future median voter imply that both fertility subsidies and SS taxes are lower than otherwise. In other respects, we find that fertility subsidies can sustain social security in cases where it would otherwise have to be abandoned, mimicking the effects of other, better understood

[6] For alternative parameterizations, we found that the ranking of taxes across models remains unchanged, but the concavity of the MPE with fertility subsidies is not guaranteed.

policy tools such as public education and migration.

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A Appendix

A.1 The choice of taxes is independent of individual endowments

Once the SS system has been continued, tax rates are chosen by solving:

$$\max_{\{\tau_t^f,\tau_{t+1}^{ss}\}} \quad (1-\tau_t^{ss})\alpha_i + \tfrac{\theta\beta\gamma\tau_{t+1}^{ss}}{(1-\tau_t^f)b} - \tfrac{\gamma}{1-\tau_t^f} \quad +\gamma\ln\frac{\gamma}{(1-\tau_t^f)b} \tag{30}$$

$$s.t. \quad -\tau_{t+1}^{ss}\alpha_{t+1}^m + \tfrac{\theta\beta\gamma\hat{\tau}_{t+2}^{ss}}{(1-\tau_{t+1}^f)b}$$

$$-\tfrac{\gamma}{1-\tau_{t+1}^f} + \gamma + \gamma\ln\tfrac{1}{1-\tau_{t+1}^f} = 0 \quad \text{if } \tau_t^f > 1-\gamma/b \tag{31}$$

$$\tau_{t+1}^{ss} \leq 1 \qquad \text{otherwise} \tag{32}$$

Where α_{t+1}^m is a function of n_t (equation (22). This problem is equivalent to the modified problem where $(1-\tau_t^{ss})w\alpha_i$ is eliminated from the objective function:

$$\max_{\{\tau_t^f,\tau_{t+1}^{ss}\}} \quad \tfrac{\theta\beta\gamma\tau_{t+1}^{ss}}{(1-\tau_t^f)b} - \tfrac{\gamma}{1-\tau_t^f} \quad +\gamma\ln\frac{\gamma}{(1-\tau_t^f)b} \tag{33}$$

$$s.t. \quad -\tau_{t+1}^{ss}\alpha_{t+1}^m + \tfrac{\theta\beta\gamma\hat{\tau}_{t+2}^{ss}}{(1-\tau_{t+1}^f)b}$$

$$-\tfrac{\gamma}{1-\tau_{t+1}^f} + \gamma + \gamma\ln\tfrac{1}{1-\tau_{t+1}^f} = 0 \quad \text{if } \tau_t^f > 1-\gamma/b \tag{34}$$

$$\tau_{t+1}^{ss} \leq 1 \qquad \text{otherwise} \tag{35}$$

Because the endowment (α_i) does not play a role in this problem, the solution cannot be a function of it.

A.2 Derivation of the MPE for the model with myopic voters

From (24) we obtain:

$$\tau_t^f = \tau_{t+1}^{ss}\frac{\theta\beta}{b} \tag{36}$$

Note that equation (25) can be written as

$$-\tau_{t+1}^{ss}\{\underline{\alpha} + \tfrac{(\overline{\alpha}-\underline{\alpha})}{2}(1-(1-\tau_t^f)\tfrac{b}{\gamma})\} \tag{37}$$

$$+\tfrac{\gamma}{1-\tau_{t+1}^f}(\tfrac{\theta\beta}{b}\tau_{t+2}^{ss} - \tau_{t+1}^f) + \gamma\ln\tfrac{1}{1-\tau_{t+1}^f} = 0$$

The second term of this expression is zero, which comes from leading expression (36) one period. Using the identity in (28), equation (37) becomes

$$\tau_{t+1}^{ss}\widehat{\alpha_{m,t+1}} = \gamma \ln\left(\frac{1}{1-\frac{\theta\beta}{b}\tau_{t+2}^{ss}}\right) \tag{38}$$

This leads to

$$\tau_{t+2}^{ss} = \frac{b}{\theta\beta}\{1-\exp\{-\frac{\tau_{t+1}^{ss}\widehat{\alpha_{m,t+1}}}{\gamma}\}\} \tag{39}$$

The MPE in (26) is this same expression lagged one period. The function characterizing the evolution of fertility subsidies can be obtained from (39) and (36).

Figure 1: Timing of decisions

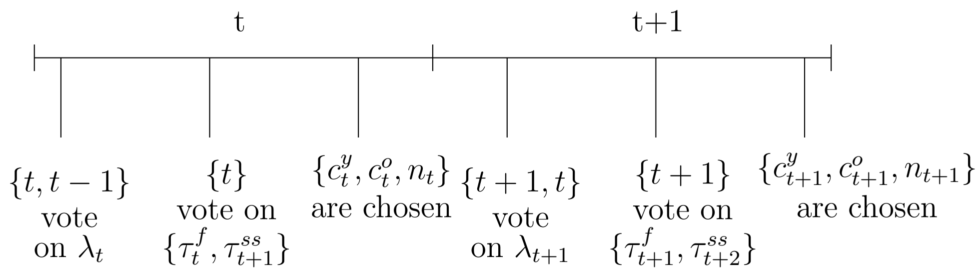


Figure 2: Choice of tax rates

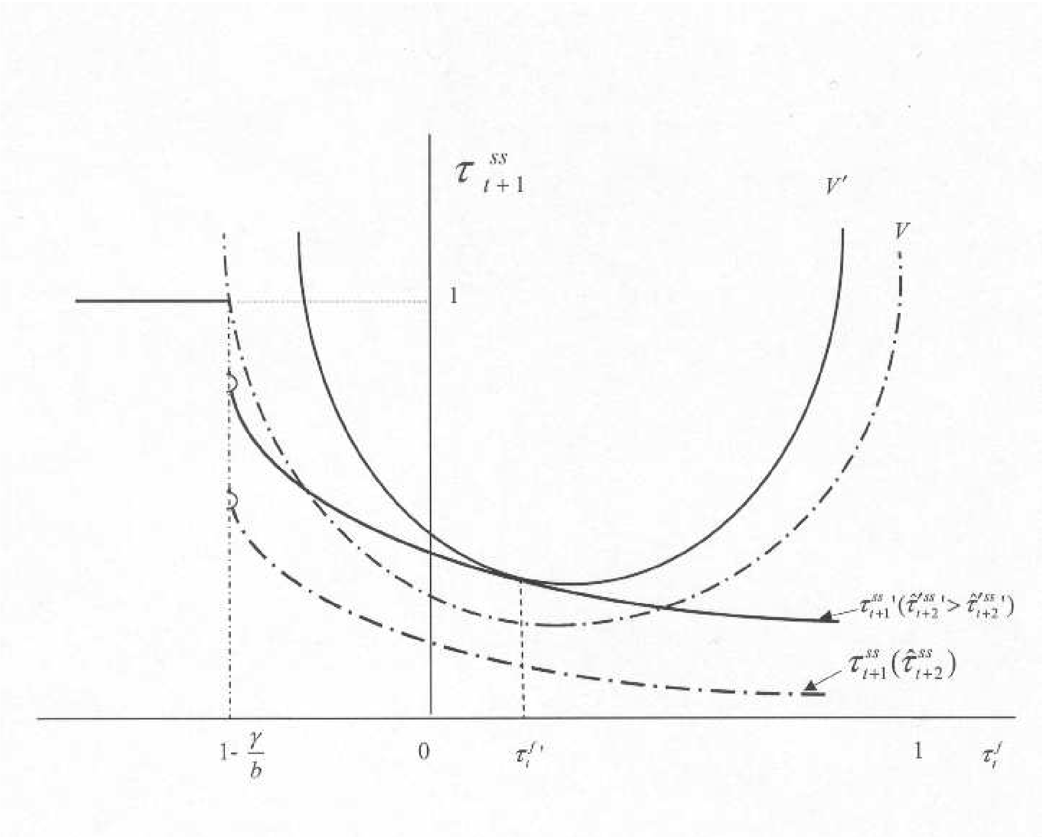


Figure 3: Who is the median voter

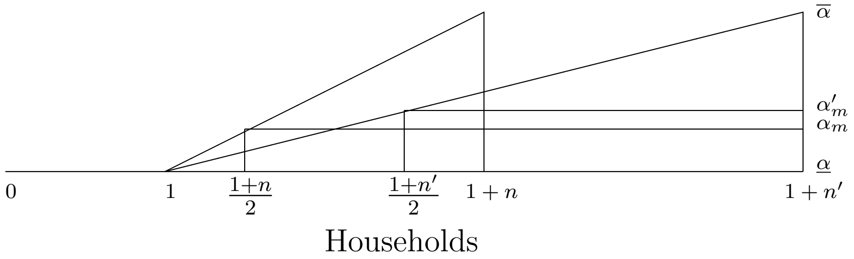
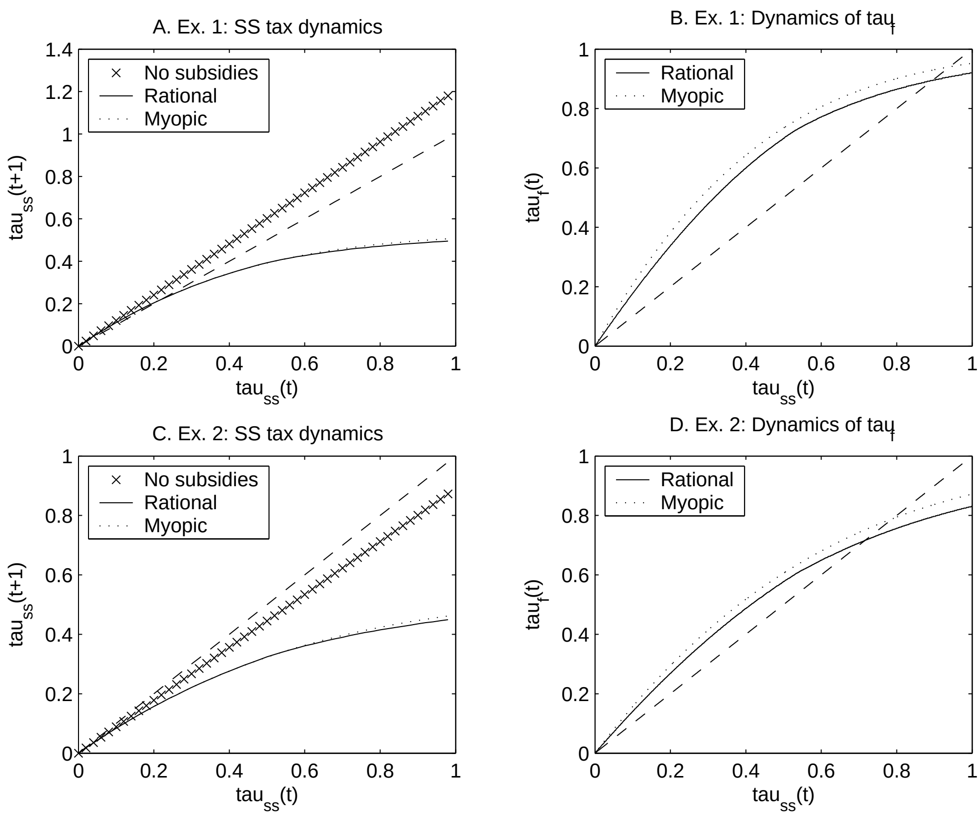


Figure 4: Tax rate dynamics



$$Parameters\ Example\ 1: \quad \{\beta = .15, b = .12, \gamma = .56, \overline{\alpha} = 2.6, \underline{\alpha} = .4\}$$
$$Parameters\ Example\ 2: \quad \{\beta = .15, b = .12, \gamma = .8, \overline{\alpha} = 2.6, \underline{\alpha} = .4\}$$